Grade Level/Course: Grade 7, Grade 8 & Algebra I

Lesson/Unit Plan Name: Quadratic Equations, Multiple Representations & What We Know

Rationale/Lesson Abstract: The idea of this lesson is to have students make connections between the multiple ways that a quadratic equation can be represented and what we know about these equations. This lesson will cover quadratic equations, their concavity, intercepts, axis of symmetry, vertices, maximums and minimums, and number of real solutions. After filling out a chart with the students, we will have the students fill in their own charts beginning with a variety of different information. This process will be repeated so that students will have a chance to make connections between verbal descriptions, equations, a table of values, graphs and the information we know. Note that the worked out "our turn" and "your turn" charts can also be used as a matching activity. Also note that how students write out their verbal descriptions, find their axis of symmetry and vertex (using the table or formulas), and the order in which they fill out their information may vary.

Timeframe: Can be used as a one-day, two-day, or multiple day lesson to review the various ways that a quadratic equation can be represented and what we know about these equations. The length of the lesson depends on how frequently you revisit the template and how deep you decide to go with the students.

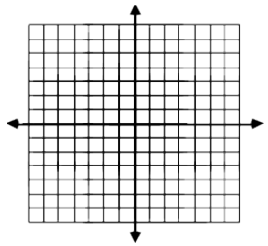
Common Core Standards: 8.F.3, A.CED.2, A.REI.4, F.IF.4.7a.8a.9

California State Standards: 7 AF 3.1, Alg. 21.0

Instructional Resources/Materials: Copies of Our Turn/Your Turn Templates (or have students create their own). For matching activity, make copies of the "worked out" versions and cut them up into columns.

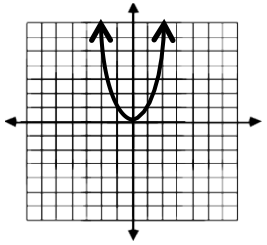
Our Turn

Given Verbal Description

Verbal Description	Equation	Table of Values	Graph	Things We Know
A number y is the square of a number x.		Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ x-intercept(s): _____ y-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Solutions when $y = 0$? _____

Our Turn! (worked out)

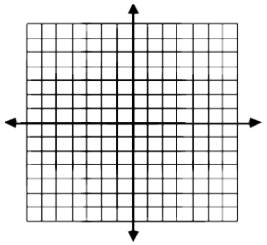
Given Verbal Description

Verbal Description	Equation	Table of Values	Graph	Things We Know
A number y is the square of a number x.	$y = x^2$	See below		Quadratic Equation Concave Up x-intercept: (0,0) y-intercept: (0,0) Axis of Symmetry $x = 0$ Vertex: (0,0) Minimum One solution

Input (x)	Equation $y = x^2$	Output (y)	(Input, Output) (x, y)
-2	$y = (-2)^2$ $y = 4$	4	$(-2,4)$
-1	$y = (-1)^2$ $y = 1$	1	$(-1,1)$
0	$y = (0)^2$ $y = 0$	0	$(0,0)$
1	$y = (1)^2$ $y = 1$	1	$(1,1)$
2	$y = (2)^2$ $y = 2$	4	$(2,4)$

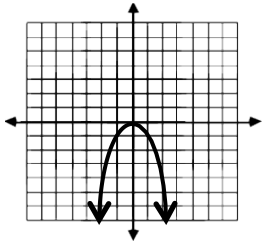
Your Turn!!

Given Equation

Verbal Description	Equation	Table of Values	Graph	Things We Know
	$y = -x^2$	Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ *x*-intercept(s): _____ *y*-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Roots? _____

Your Turn!! (worked out)

Given Equation

<table>
<tr><th>Verbal Description</th><th>Equation</th><th>Table of Values</th><th>Graph</th><th>Things We Know</th></tr>
<tr>
<td>A number y is the opposite of a number x to the second power.</td>
<td>$y = -x^2$</td>
<td>
<table>
<tr><th>Input (x)</th><th>Equation $y = -x^2$</th><th>Output (y)</th><th>(Input, Output) (x, y)</th></tr>
<tr><td>-2</td><td>$y = -(-2)^2$
$y = -4$</td><td>-4</td><td>$(-2, -4)$</td></tr>
<tr><td>-1</td><td>$y = -(-1)^2$
$y = -1$</td><td>-1</td><td>$(-1, -1)$</td></tr>
<tr><td>0</td><td>$y = -(0)^2$
$y = 0$</td><td>0</td><td>$(0,0)$</td></tr>
<tr><td>1</td><td>$y = -(1)^2$
$y = -1$</td><td>-1</td><td>$(1, -1)$</td></tr>
<tr><td>2</td><td>$y = -(2)^2$
$y = -4$</td><td>-4</td><td>$(2, -4)$</td></tr>
</table>
</td>
<td>  </td>
<td>Quadratic Equation

Concave Down

x-intercept: (0,0)

y-intercept: (0,0)

Axis of Symmetry $x = 0$

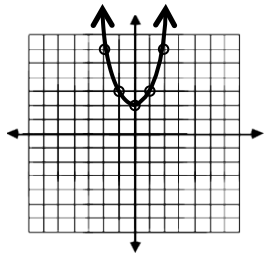
Vertex: (0,0)

Maximum

One solution</td>
</tr>
</table>

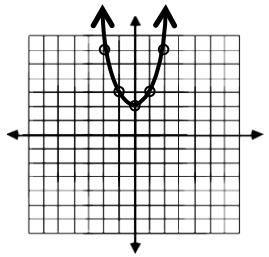
Your Turn!!

Given Graph

Verbal Description	Equation	Table of Values	Graph	Things We Know
		Input (x) | Equation | Output (y) | (Input, Output) (x, y)	$(-2,6), (-1,3), (0,2), (1,3), (2,6)$ 	Linear, quadratic, cubic, …? _____ Concavity? _____ x-intercept(s): _____ y-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Solutions when $y = 0$? _____

Your Turn!! (worked out)

Given Graph

<table>
<tr><th>Verbal Description</th><th>Equation</th><th>Table of Values</th><th>Graph</th><th>Things We Know</th></tr>
<tr>
<td>A number y is equal to the sum of a number x squared and two.</td>
<td>$y = x^2 + 2$</td>
<td>
<table>
<tr><th>Input (x)</th><th>Equation $y = x^2 + 2$</th><th>Output (y)</th><th>(Input, Output) (x, y)</th></tr>
<tr><td>-2</td><td>$y = (-2)^2 + 2$
$y = 4 + 2$
$y = 6$</td><td>6</td><td>$(-2,6)$</td></tr>
<tr><td>-1</td><td>$y = (-1)^2 + 2$
$y = 1 + 2$
$y = 3$</td><td>3</td><td>$(-1,3)$</td></tr>
<tr><td>0</td><td>$y = (0)^2 + 2$
$y = 0 + 2$
$y = 2$</td><td>2</td><td>$(0,2)$</td></tr>
<tr><td>1</td><td>$y = (1)^2 + 2$
$y = 1 + 2$
$y = 3$</td><td>3</td><td>$(1,3)$</td></tr>
<tr><td>2</td><td>$y = (2)^2 + 2$
$y = 4 + 2$
$y = 6$</td><td>6</td><td>$(2,6)$</td></tr>
</table>
</td>
<td>$(-2,6), (-1,3), (0,2), (1,3), (2,6)$
  </td>
<td>Quadratic Equation

Concave Up

x-intercept: none

y-intercept: $(0,2)$

Axis of Symmetry $x = 0$

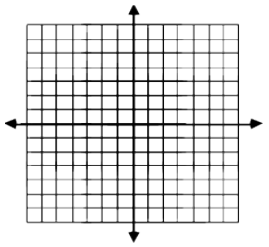
Vertex: $(0,2)$

Minimum

No solutions</td>
</tr>
</table>

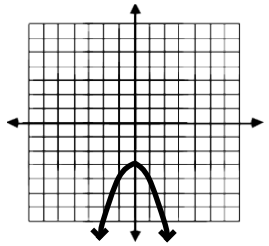
Your Turn!!

Given Things We Know

Verbal Description	Equation	Table of Values	Graph	Things We Know
		Input (x) | Equation | Output (y) | (Input, Output) (x, y)	 Points at: $(-1, -4)$ and $(-2, -7)$	Quadratic Equation Concave Down x-intercept: none y-intercept: $(0, -3)$ Axis of Symmetry $x = 0$ Vertex: $(0, -3)$ Maximum No solutions

Your Turn!! (worked out)

Given Things We Know

<table>
<tr><th>Verbal Description</th><th>Equation</th><th>Table of Values</th><th>Graph</th><th>Things We Know</th></tr>
<tr>
<td>The opposite of a number x to the second power subtracted by three is equal to a number y.</td>
<td>$y = -x^2 - 3$</td>
<td>
<table>
<tr><th>Input
(x)</th><th>Equation
$y = -x^2 - 3$</th><th>Output
(y)</th><th>(Input, Output)
(x, y)</th></tr>
<tr><td>-2</td><td>$y = -(-2)^2 - 3$
$y = -4 - 3$
$y = -7$</td><td>-7</td><td>$(-2, -7)$</td></tr>
<tr><td>-1</td><td>$y = -(-1)^2 - 3$
$y = -1 - 3$
$y = -4$</td><td>-4</td><td>$(-1, -4)$</td></tr>
<tr><td>0</td><td>$y = -(0)^2 - 3$
$y = 0 - 3$
$y = -3$</td><td>-3</td><td>$(0, -3)$</td></tr>
<tr><td>1</td><td>$y = -(1)^2 - 3$
$y = -1 - 3$
$y = -4$</td><td>-4</td><td>$(1, -4)$</td></tr>
<tr><td>2</td><td>$y = -(2)^2 - 3$
$y = -4 - 3$
$y = -7$</td><td>-7</td><td>$(2, -7)$</td></tr>
</table>
</td>
<td>  </td>
<td>Quadratic Equation

Concave Down

x-intercept: none

y-intercept: $(0, -3)$

Axis of Symmetry $x = 0$

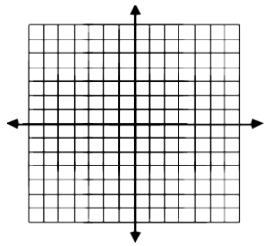
Vertex: $(0, -3)$

Maximum

No solutions</td>
</tr>
</table>

Our Turn

Given Verbal Description

Verbal Description	Equation	Table of Values	Graph	Things We Know
A number y is twice the square of a number x.		Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ x-intercept(s): _____ y-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Roots? _____

Our Turn! (worked out)

Given Verbal Description

<table>
<tr><th>Verbal Description</th><th>Equation</th><th>Table of Values</th><th>Graph</th><th>Things We Know</th></tr>
<tr>
<td>A number y is twice the square of a number x.</td>
<td>$y = 2x^2$</td>
<td>
<table>
<tr><th>Input (x)</th><th>Equation $y = 2x^2$</th><th>Output (y)</th><th>(Input, Output) (x, y)</th></tr>
<tr><td>-2</td><td>$y = 2(-2)^2$
$y = 2(4)$
$y = 8$</td><td>8</td><td>$(-2,8)$</td></tr>
<tr><td>-1</td><td>$y = 2(-1)^2$
$y = 2(1)$
$y = 2$</td><td>2</td><td>$(-1,2)$</td></tr>
<tr><td>0</td><td>$y = 2(0)^2$
$y = 2(0)$
$y = 0$</td><td>0</td><td>$(0,0)$</td></tr>
<tr><td>1</td><td>$y = 2(1)^2$
$y = 2(1)$
$y = 2$</td><td>2</td><td>$(1,2)$</td></tr>
<tr><td>2</td><td>$y = 2(2)^2$
$y = 2(4)$
$y = 8$</td><td>8</td><td>$(2,8)$</td></tr>
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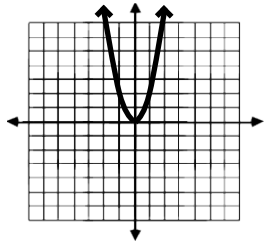
<td>  </td>
<td>Quadratic Equation

Concave Up

x-intercept: (0,0)

y-intercept: (0,0)

Axis of Symmetry $x = 0$

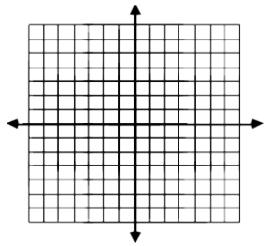
Vertex: (0,0)

Minimum

One solution</td>
</tr>
</table>

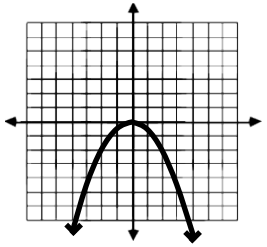
Our Turn

Given Equation

Verbal Description	Equation	Table of Values	Graph	Things We Know
	$y = -\frac{1}{2}x^2$	Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ *x*-intercept(s): _____ *y*-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Solutions when $y = 0$? _____

Our Turn! (worked out)

Given Equation

<table>
<tr>
<th>Verbal Description</th>
<th>Equation</th>
<th>Table of Values</th>
<th>Graph</th>
<th>Things We Know</th>
</tr>
<tr>
<td>A number y is negative one half multiplied by the square of a number x.</td>
<td>$y = -\frac{1}{2}x^2$</td>
<td>
<table>
<tr><th>Input
(x)</th><th>Equation
$y = -\frac{1}{2}x^2$</th><th>Output
(y)</th><th>(Input, Output)
(x, y)</th></tr>
<tr><td>-2</td><td>$y = -\frac{1}{2}(-2)^2$
$y = -\frac{1}{2}(4)$
$y = -2$</td><td>-2</td><td>$(-2, -2)$</td></tr>
<tr><td>-1</td><td>$y = -\frac{1}{2}(-1)^2$
$y = -\frac{1}{2}(1)$
$y = -\frac{1}{2}$</td><td>$-\frac{1}{2}$</td><td>$\left(-1, -\frac{1}{2}\right)$</td></tr>
<tr><td>0</td><td>$y = -\frac{1}{2}(0)^2$
$y = -\frac{1}{2}(0)$
$y = 0$</td><td>0</td><td>$(0,0)$</td></tr>
<tr><td>1</td><td>$y = -\frac{1}{2}(1)^2$
$y = -\frac{1}{2}(1)$
$y = -\frac{1}{2}$</td><td>$-\frac{1}{2}$</td><td>$\left(1, -\frac{1}{2}\right)$</td></tr>
<tr><td>2</td><td>$y = -\frac{1}{2}(2)^2$
$y = -\frac{1}{2}(4)$
$y = -2$</td><td>-2</td><td>$(2, -2)$</td></tr>
</table>
</td>
<td>  </td>
<td>Quadratic Equation

Concave Down

x-intercept:
(0,0)

y-intercept:
(0,0)

Axis of Symmetry
$x = 0$

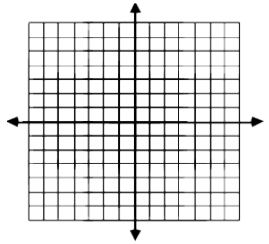
Vertex:
(0,0)

Maximum

One solution</td>
</tr>
</table>

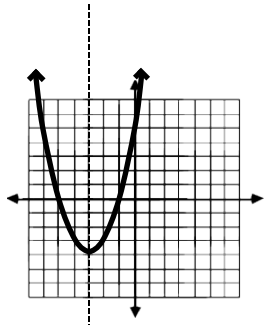
Our Turn!!

Given Quadratic Equation:
Solve by Factoring & The Zero Product Property

Verbal Description	Equation	Table of Values	Graph	Things We Know
	$0 = x^2 + 6x + 5$	Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ *x*-intercept(s): _____ *y*-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Roots? _____

Our Turn!! (worked out)

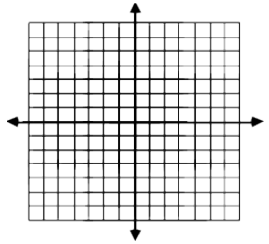
Given Quadratic Equation:
Solve by Factoring & The Zero Product Property

Verbal Description	Equation	Table of Values	Graph	Things We Know
A number y is equal to a number x squared plus the product of six and a number x plus five.	$0 = x^2 + 6x + 5$ $0 = (x + 5)(x + 1)$ $(x + 5) = 0$ or $(x + 1) = 0$ $x = -5$ or $x = -1$	See table below	 Axis of Symmetry (Note: This is the first example where the axis of symmetry is not the y-axis)	Quadratic Equation Concave Up x-intercepts: $(-5,0), (-1,0)$ y-intercept: $(0,5)$ Axis of Symmetry $x = -3$ Vertex: $(-3, -4)$ Minimum Two solutions

Input (x)	Equation $y = x^2 + 6x + 5$	Output (y)	(Input, Output) (x, y)
-2	$y = (-2)^2 + 6(-2) + 5$ $y = 4 - 12 + 5$ $y = -3$	-3	$(-2, -3)$
-1	$y = (-1)^2 + 6(-1) + 5$ $y = 1 - 6 + 5$ $y = 0$	0	$(-1,0)$
0	$y = (0)^2 + 6(0) + 5$ $y = 0 + 0 + 5$ $y = 5$	5	$(0,5)$
1	$y = (1)^2 + 6(1) + 5$ $y = 1 + 6 + 5$ $y = 12$	12	$(1,12)$
2	$y = (2)^2 + 6(2) + 5$ $y = 4 + 12 + 5$ $y = 21$	21	$(2,21)$

Our Turn!!

Given Quadratic Equation:
Solve by Quadratic Formula

Verbal Description	Equation	Table of Values	Graph	Things We Know
	$0 = x^2 + 2x - 2$	Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ *x*-intercept(s): _____ *y*-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Solutions when *y* = 0? _____

Our Turn!! (worked out)

Given Quadratic Equation: Solve by Quadratic Formula

Verbal Description

A number y is equal to a number x squared plus twice the number x subtracted by two.

Equation

$0 = x^2 + 2x - 2$

$a = 1, b = 2, c = -2$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4+8}}{2}$

$x = \frac{-2 \pm \sqrt{12}}{2}$

$x = \frac{-2 \pm 2\sqrt{3}}{2}$

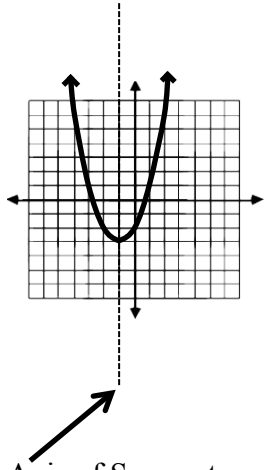
$x = -1 \pm \sqrt{3}$

$x = -1 + \sqrt{3}$ $x = -1 - \sqrt{3}$

Table of Values

Input (x)	Equation $y = x^2 + 2x - 2$	Output (y)	(Input, Output) (x, y)
-2	$y = (-2)^2 + 2(-2) - 2$ $y = 4 - 4 - 2$ $y = -2$	-2	$(-2, -2)$
-1	$y = (-1)^2 + 2(-1) - 2$ $y = 1 - 2 - 2$ $y = -3$	-3	$(-1, -3)$
0	$y = (0)^2 + 2(0) - 2$ $y = 0 + 0 - 2$ $y = -2$	-2	$(0, -2)$
1	$y = (1)^2 + 2(1) - 2$ $y = 1 + 2 - 2$ $y = 1$	1	(1,1)
2	$y = (2)^2 + 2(2) - 2$ $y = 4 + 4 - 2$ $y = 6$	6	(2,6)

Graph



Axis of Symmetry

Things We Know

Quadratic Equation

Concave Up

x-intercepts: $(-1 + \sqrt{3}, 0)$, $(-1 - \sqrt{3}, 0)$

y-intercept: $(0, -2)$

Axis of Symmetry $x = -1$

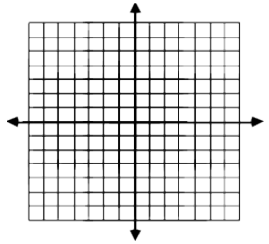
Vertex: $(-1, -3)$

Minimum

Two solutions

Our Turn!!

Given Quadratic Equation:
Solve by Completing the Square

Verbal Description	Equation	Table of Values	Graph	Things We Know
	$x^2 + 4x + 2 = 0$	Input (x) | Equation | Output (y) | (Input, Output) (x, y)		Linear, quadratic, cubic, …? _____ Concavity? _____ *x*-intercept(s): _____ *y*-intercept(s): _____ Axis of Symmetry: ___ Vertex: _____ Maximum or Minimum? _____ # of Real Roots? _____

Our Turn!! (worked out)

Given Quadratic Equation: Solve by Completing the Square

Verbal Description

A number x squared plus the product of four and a number x plus two is equal to a number y.

Equation

$x^2 + 4x + 2 = 0$

$x^2 + 4x = -2$

$x^2 + 4x + (2)^2 = -2 + 4$

$(x + 2)^2 = 2$

$\sqrt{(x + 2)^2} = \sqrt{2}$

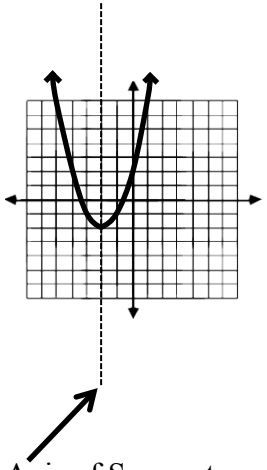
$x + 2 = \pm\sqrt{2}$

$x = -2 \pm \sqrt{2}$

Table of Values

Input (x)	Equation $y = x^2 + 4x + 2$	Output (y)	(Input, Output) (x, y)
-2	$y = (-2)^2 + 4(-2) + 2$ $y = 4 - 8 + 2$ $y = -2$	-2	$(-2, -2)$
-1	$y = (-1)^2 + 4(-1) + 2$ $y = 1 - 4 + 2$ $y = -1$	-1	$(-1, -1)$
0	$y = (0)^2 + 4(0) + 2$ $y = 0 + 0 + 2$ $y = 2$	2	(0,2)
1	$y = (1)^2 + 4(1) + 2$ $y = 1 + 4 + 2$ $y = 7$	7	(1,7)
2	$y = (2)^2 + 4(2) + 2$ $y = 4 + 8 + 2$ $y = 14$	14	(2,14)

Graph



Things We Know

Quadratic Equation

Concave Up

x-intercepts: $(-2 + \sqrt{2}, 0)$, $(-2 - \sqrt{2}, 0)$

y-intercept: (0,2)

Axis of Symmetry $x = -2$

Vertex: $(-2, -2)$

Minimum

Two solutions